100 between the two luminous luxes. We note that the stellar magnitude increases when the star turns fainter. The absolute scale is calibrated such that the brightest stars are having magnitudes about unity, and the faintest stars that are visible to the bare eye, also known as *visual limiting magnitudes*, are lying between 6 − 7mag in sufficiently dark nights. Often, poor transparencies of the atmosphere, or the vicinity to cities are restricting the limiting visual magnitudes to values below 5mag. The naked eye limiting magnitude is also a decisive parameter for the limiting magnitude achieved in combination with the binocular.

10.5.2 Astro-indices for limiting magnitudes

During extensive field studies, Ed Zarenski has determined the limiting magnitudes of a considerably large set of binoculars[8] and summarized his findings in figure 10.9. The data suggest that it is not not only the aperture that decides on the performance, but the magnification as well. The *Adler index*, proposed by Alan Adler and defined as the product of magnification and the square root of the objective diameter,

$$I_A = mD^{1/2} , \qquad \text{(Adler index)} \qquad (10.17)$$

is in close agreement with Zarenski's limiting magnitudes. For example, a 10x50 binocular would yield the Adler index of $I_{A,1} = 70.7$, while the larger 20x100 device would score $I_{A,2} = 200$, and thus a performance gain of $200/70.7 = 2.83$. Translated into magnitudes, this performance gain would then amount to

$$\Delta(\text{mag}) = \frac{\log(I_{A,2}/I_{A,1})}{\log(2.512)} = \frac{\log(2.83)}{0.4} = 1.13\text{mag} . \qquad (10.18)$$

[8] Ed Zarenski, *CN Report: Limiting Magnitude in Binoculars*, 2003, auf `www.cloudynights.com/`

Recently, Beat Fankhauser has pointed out that the scaling behavior of the Adler index would violate basic physical principles[9]: Obviously, the 20x100 binocular is collecting four times as much light as the 10x50 device. Assuming same quality features such as transmission, that factor of four would apply to the flux of light exiting the binocular, too. Additionally, both binoculars are having identical exit pupil diameters, so that on the perception side of the equation all other parameters would remain the same. Naturally, the performance gain of the 20x100 over the 10x50 binocular should exactly amount to four, and the gain in limiting magnitudes therefore 1.51mag, instead of 1.13mag as predicted by the Adler index. Fankhauser correctly observed that any performance index would have to satisfy the scaling law

$$I = (m^a D^b)^{2/(a+b)} , \qquad (10.19)$$

with an arbitrary choice of the two exponents (a, b), in order to conserve the energy flux that passes through the instrument. He suggested the choice of $a = 2$ and $b = 1$, to achieve a dominance of the magnification over the aperture, just as the Adler index, but this time with the correct scaling law

$$I_F = (m^2 D)^{2/3} . \qquad \text{(Fankhauser index)} \quad (10.20)$$

Another alternative approach, commonly known as *visibility factor*[10] and suggested by Roy Bishop, is weighting both magnitude and aperture evenly, with the choice $a = 1$ and $b = 1$, yielding

$$I_V = mD . \qquad \text{(Bishop index)} \qquad (10.21)$$

But which of these indices is most accurate? Is there a way to approach this problem from a rather scientific angle, based on models of human perception? As it turns out, Berek's model may be applied to the limiting magnitudes of stars.

[9] Beat Fankhauser, *Eine neue Leistungsgröße für Ferngläser*, ORION, Nr. 387, 2/2015.

[10] Lambert Spix, *Fern-Seher*, Oculum-Verlag Erlangen, S. 24 (2013)