generally remain quite moderate, and it is hard to decide which of the astro indices is performing best. This fact is all the more puzzling because we have already shown that the Adler index is violating fundamental principles and thus should not be expected to remain accurate. The indices of Fankhauser and Bishop are having different scaling exponents and thus should deviate in their predictions, too.

An explanation for the surprising similarity of the various astro indices is found after the introduction of the exit pupil diameter $d = D/m$ into the general index (10.19), yielding

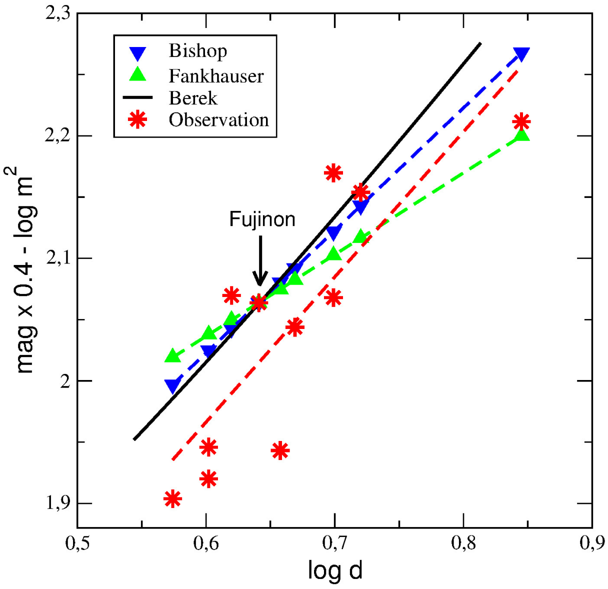
$$I = m^2 d^{2b/(a+b)} \,. \tag{10.28}$$

Expressed in this way, the entire set of possible indices, which is spanned over the parameter space of a and b, exclusively differs in their exponents of the exit pupil diameter: $2b/(a+b) = 1$ (Bishop), or $2/3$ (Fankhauser). We then derive

$$\begin{aligned} \log\left(\frac{I}{m^2}\right) &= \text{mag(I)} \log(2.512) - \log\left(m^2\right) \\ &= \log\left(d^{2b/(a+b)}\right) \,, \end{aligned} \tag{10.29}$$

and the rescaled plot mag(I) $\cdot$ 0.4 versus $\log d$ then contains lines with the slopes $2b/(a+b)$. This plot is shown in figure 10.11: The magnitudes evaluated with the Bishop index (blue) are aligned along the curve of slope 1, the predictions of Fankhauser (green) of slope $2/3$, and Berek's predictions are forming a curve with an approximate slope of 1.25 (black curve).

In this plot, the indices do in fact yield distinguishable predictions, but as soon as the observation data are added (red stars), it becomes clear why they do not discriminate between the models: They are scattering significantly, and no well defined slope is visible. A linear regression would yield the slope $\approx 1.18 \pm 0.25$ (dashed red line), including a significant uncertainty. In this plot, the observation data are hardly decisive, because the range of exit pupils was limited to $3.75\text{mm} \leq d \leq 7\text{mm}$, which, in combination with the scatter, is insufficient yield a well defined power law of the form $\sim d^x$. The accuracy of the observation data would thus have to be increased significantly to allow for a clean discrimination between the different astro indices – a difficult task, considering systematic quality differences between the samples. On the other hand, Figure 10.10 indicates that, for practical purposes, the accuracy of both observations and theoretical models are actually reasonably high, and any one of the existing astro indices would be sufficiently accurate. Thus, the simplest one, the Bishop index, might in fact be preferable.



10.11

Scaled limiting magnitudes as a function of the exit pupil diameter. The wide scatter of the observation data (red stars; linear interpolation shown as dashed red line) prohibits a reliable discrimination between the various theoretical models.